Chapter 10

Architectural Issues in SISO Control

This chapter considers 3 related issues namely:-

(1) Exact disturbance compensation and set point tracking (*leading to the Internal Model Principle*)

(2) Use of extra measured information about disturbances (*leading to disturbance feedforward control*)

(3) Use of additional internal measurements (*leading to cascade control*)

These are examples of architectural issues in control system design.

Indeed, one of the major tools available to the control system designer is to be able to adjust the control system architecture so as to achieve given performance objectives. Feedforward and Cascade control are prime examples of architectural changes which can significantly effect achieved performance.

The chapter contains an illustration of the positive influence, architectural issues can have by revisiting the "*Hold-Up Effect*" in Reversing Cold Rolling Mills discussed in Chapter 8.

1. Exact Disturbance Compensation via Internal Model Control

Our previous analysis has focused on basic feedback loop properties and feedback controller synthesis. Here we will extend the scope of the analysis to focus on further architectural issues which are aimed at achieving exact compensation of certain types of deterministic disturbances and exact tracking of particular reference signals.

Models for Deterministic Disturbances and References

The particular signals of interest here are those that can be described as the output of a linear dynamic system having zero input and certain specific initial conditions. The simplest example of such a signal is a constant, which can be described by the model

$$\dot{x}_d = 0 \quad ; \quad x_d(0) \quad \text{given}$$

The generalization of this idea includes any disturbance that can be described by a differential equation of the form:

$$\frac{d^q d_g(t)}{dt^q} + \sum_{i=0}^{q-1} \gamma_i \frac{d^i d_g(t)}{dt^i} = 0$$

The above model leads to the following expression for the Laplace transform of the disturbance:

$$D_g(s) = \frac{N_d(s)x_d(0)}{\Gamma_d(s)}$$

where $\Gamma_d(s)$ is the disturbance generating polynomial defined by

$$\Gamma_d(s) \triangleq s^q + \sum_{i=0}^{q-1} \gamma_i s^i$$

Example 10.1

A disturbance takes the following form

$$d_g(t) = K_1 + K_2 \sin(3t + K_3)$$

where K_1, K_2 and K_3 are constants. Then the generating polynomial is given by

$$\Gamma_d(s) = s(s^2 + 9)$$

Note that K_1, K_2 and K_3 are related to the initial state, $x_d(0)$, in the state space model.

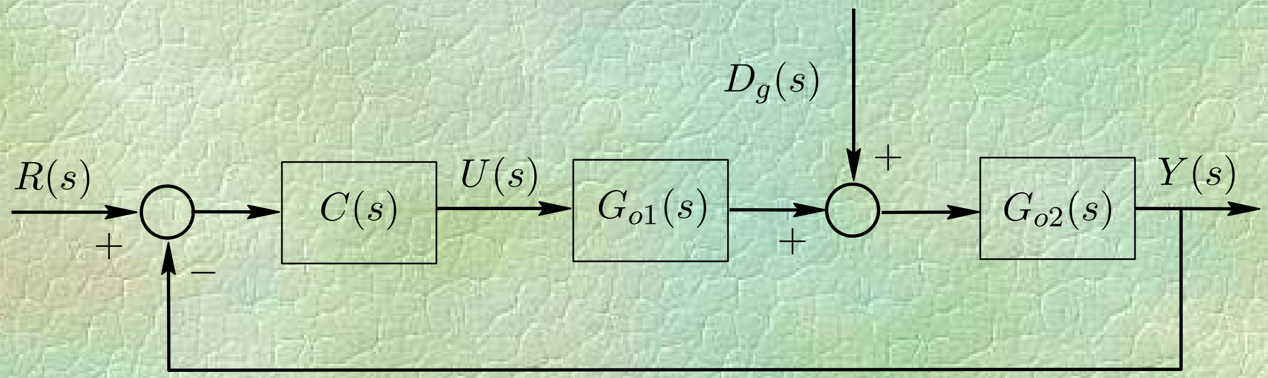
Internal Model Principle for Disturbance

Disturbance Entry Points. For a nominal model $G_0(s)$ with input $U(s)$ and output $Y(s)$, we will assume that the disturbance $D_g(s)$ acts on the plant at some intermediate point, i.e. we model the output as follows:

$$Y(s) = G_{o2}(s)(G_{o1}(s)U(s) + D_g(s)) \qquad \text{where} \qquad G_o(s) = G_{o1}(s)G_{o2}(s)$$

This is illustrated on the next figure.

Figure 10.1: *Control loop with a generalized disturbance*



Steady State Disturbance Compensation

We note that for the generalized disturbance description given above, and assuming closed loop stability, the nominal model output and controller output are given respectively by

$$Y(s) = S_o(s)G_{o2}(s)D_g(s)$$

$$U(s) = -S_{uo}G_{o2}(s)D_g(s) = \frac{T_o(s)}{G_{o1}(s)}D_g(s)$$

From the first equation we observe that the effect of the disturbance on the model output vanishes asymptotically when the polynomial $\Gamma_d(s)$ is a factor in the numerator of $S_0(s)G_{02}(s)$.

Specifically,

$$
\begin{aligned}
S_0 G_{02} &= \left(\frac{1}{1 + G_0 C} \right) G_{02} \\
&= \left(\frac{1}{1 + G_{01} G_{02} C} \right) G_{02} \\
&= \frac{A_1 B_2 L}{A_1 A_2 L + B_1 B_2 P}
\end{aligned}
$$

where

$$
G_{01} = \frac{B_1}{A_1}; \quad G_{02} = \frac{B_2}{A_2}; \quad C = \frac{P}{L}
$$

Now exact disturbance compensation occurs if the disturbance generating polynomial appears in the numerator of S_0G_{02}, i.e. in A_1, B_2 or L. Under these conditions, the steady state response is seen to be

$$y_\infty = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sS_0 G_{02} D_g = 0$$

For input disturbances, we require that the disturbance generating polynomial appear in the numerator of S_0.

We thus conclude:

A sufficient condition for steady state disturbance compensation is that the generating polynomial be included as part of the controller denominator. This is known as the *Internal Model Principle*, (IMP).

To achieve this result the controller takes the form:

$$C(s) = \frac{\overline{C}(s)}{\Gamma_d(s)}$$

where $\Gamma_d(s)$ is the appropriate disturbance generating polynomial.

Note that integral action, where $\Gamma_d(s) = s$, is a special case of this result for constant disturbances.

We next show how the above constraint can be incorporated into the standard controller synthesis procedures. In particular, we will revisit the pole-assignment strategy.

Pole Assignment

Taking $G_0 = \frac{B_0}{A_0}; \quad C = \frac{P}{\Gamma_d L}$

Then the Pole Assignment equation becomes

$$A_0 L \Gamma_d + B_0 P = A_{cl}$$

This equation can be solved in the usual way. If Γ_d has degree q, then A_{cl} needs to have degree, at least $2n$-1+q.

Example 10.2

Consider a nominal model $G_0(s) = {}^3/_{s+3}$ and an input disturbance $d_g(t) = K_1 + K_2 \sin(2t + K_3)$. It is required to build a controller $C(s)$ such that the IMP is satisfied for this class of disturbances.

We first note that $q = 3$, $\Gamma_d(s) = s(s^2 + 4)$ and $n = 1$. This means that $A_{cl}(s)$ should at least be of degree $n_c = 4$. Say we choose $A_{cl}(s) = (s^2 + 4s + 9)(s + 5)^2$. We then have that the controller should have the form

$$C(s) = \frac{\beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0}{s(s^2+4)}$$

The corresponding pole assignment equation becomes

$$s(s^2+4)(s+3)+3(\beta_3 s^3+\beta_2 s^2+\beta_1 s+\beta_0)=(s^2+4s+9)(s+5)^2$$

leading to $\beta_3 = \frac{14}{3}, \beta_2 = \frac{74}{3}, \beta_1 = \frac{190}{3}$ and $\beta_0 = 75$ (use paq.m).

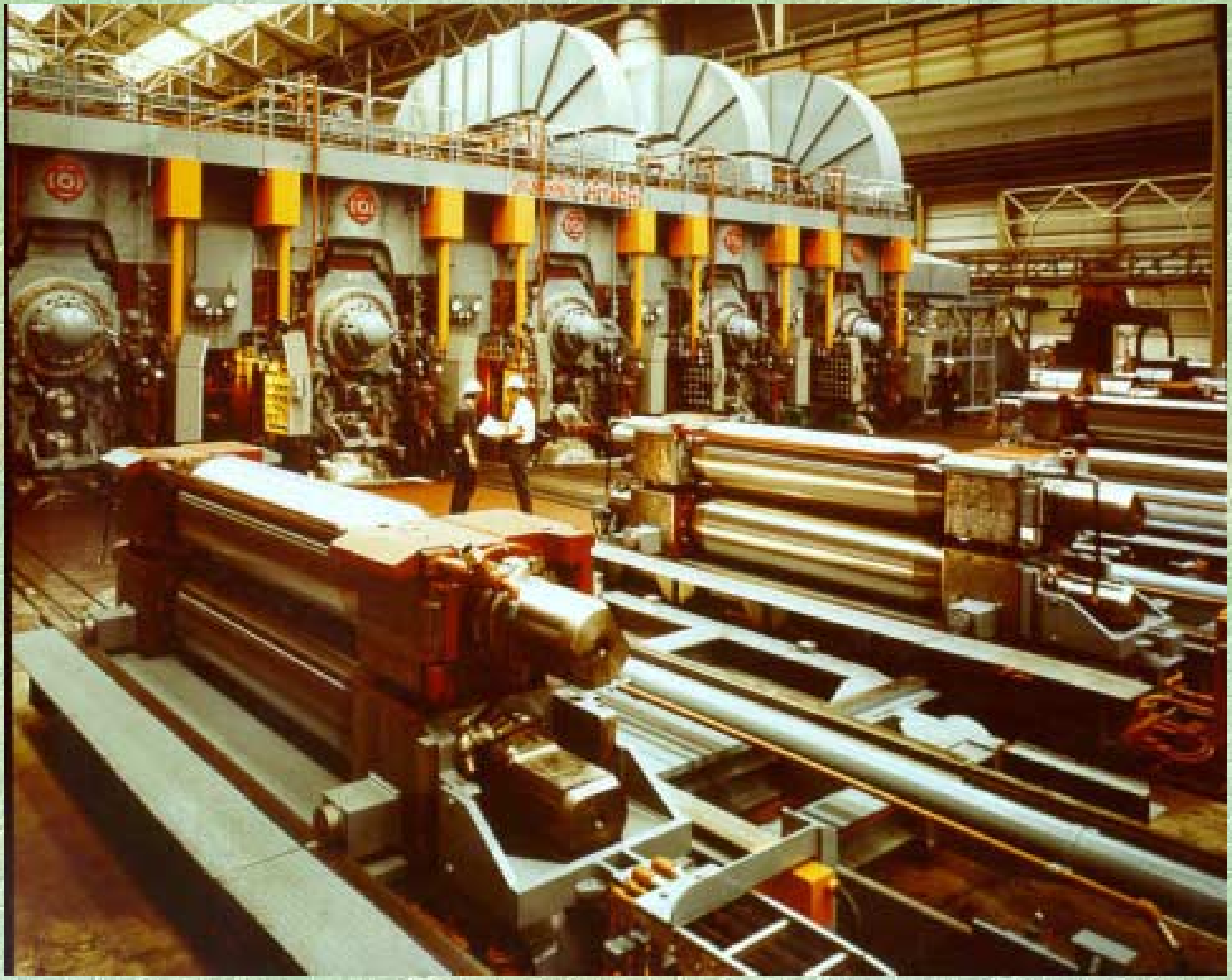
Industrial Application: Roll eccentricity compensation in rolling mills

A common technique used for gauge control in rolling mills is to infer thickness from roll force measurements. This is commonly called a **BISRA** gauge. However these measurements are affected by roll eccentricity (*which induces sinusoidal type disturbances*).

A very common strategy for dealing with this problem is to model the eccentricity components as multiple sinusoids (*ten sine waves per roll are typically used; with four rolls, this amounts to forty sinusoids*). These sinusoids can be modeled using a generating polynomial of the form

$$\Gamma_d(s) = \prod_{i=1}^{m} (s^2 + \omega_i^2)$$

Multi-stand Rolling Mill



The Internal Model Principle can then be used to remove the disturbance from the exit gauge.

An illustration of this idea is given on the book’s web page.

We next consider the problem of exactly tracking certain reference signals.

Again we assume that the reference signals can be modeled by a homogenous equation having generating polynomial $\Gamma_r(s)$.

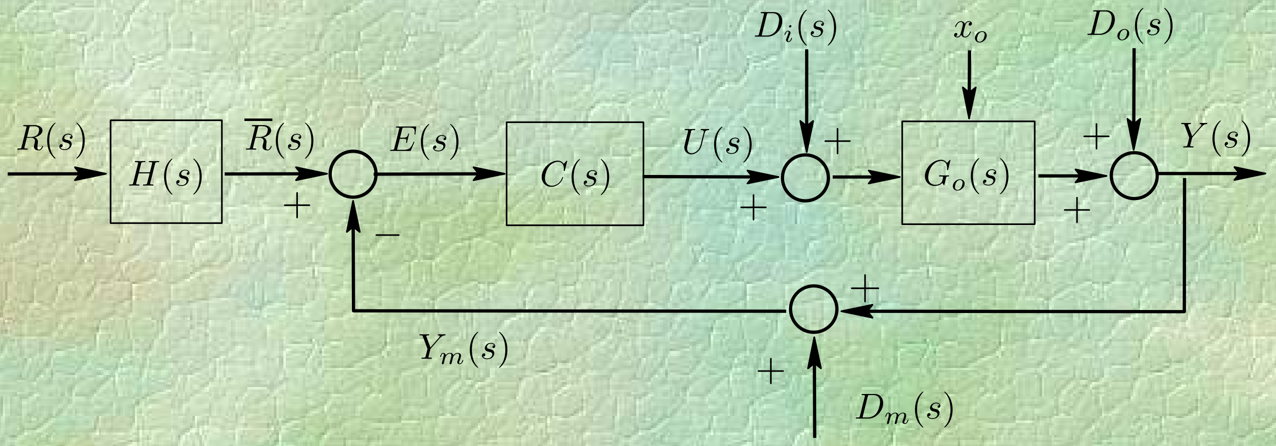
Internal Model Principle for Reference Tracking

For reference tracking, we consider the two degree of freedom architecture shown in Figure 5.2 (*see next slide*) with zero disturbances. Then the tracking performance can be quantified through the following equations:

$$Y(s) = H(s)T_o(s)R(s)$$
$$E(s) = R(s) - Y(s) = (1 - H(s)T_o(s))R(s)$$
$$U(s) = H(s)S_{uo}(s)R(s)$$

If we are to use the internal model principle for reference tracking, then it suffices to set $H(s) = 1$ and then to ensure that the reference generating polynomial is included in the denominator of $C(s)G_0(s)$.

Figure 5.2: *Two degree of freedom closed loop*



To achieve robust tracking, the reference generating polynomial must be in the denominator of the product $C(s)G_0(s)$, i.e. the Internal Model Principle also has to be satisfied for the reference. When the reference generating polynomial and the disturbance generating polynomial share some roots, then these common roots need only be included once in the denominator of $C(s)$ to simultaneously satisfy the IMP for both the reference and disturbance.

Reference Feedforward

We can use a two-degree-of-freedom architecture for reference tracking. The essential idea of reference feedforward is to use $H(s)$ to invert $T_0(s)$ at certain key frequencies, i.e. so that $H(s)T_0(s) = 1$ at the poles of the reference model (i.e. at ϵ_i, $i = 1, \ldots, n_e$). Note that, by this strategy, one can avoid using high gain feedback to bring $T_0(\epsilon_i)$ to 1. Note, however, that use of reference feedforward in this way does not give lead to perfect tracking if there is a change in the model. This contrasts with the use of the IMP which always gives exact tracking (provided stability is retained).

We will next show how extra measurements which are related to disturbances can be used to improve the transient performance achieved when compensating disturbances. This leads us to the idea of feedforward control.

2. Feedforward

The use of the IMP, as outlined above, provides complete disturbance compensation and reference tracking in steady state for certain classes of signals (e.g. constants, sinusoids, etc). However, this leaves unanswered the issue of transient performance, i.e. how the system responds during the initial phase of the response following a change in the disturbance or reference signal.

We will show how feedforward can aid this problem.

Disturbance Feedforward

We show how feedforward ideas can be applied to disturbance rejection.

A structure for feedforward from a measurable disturbance is shown in Figure 10.2.

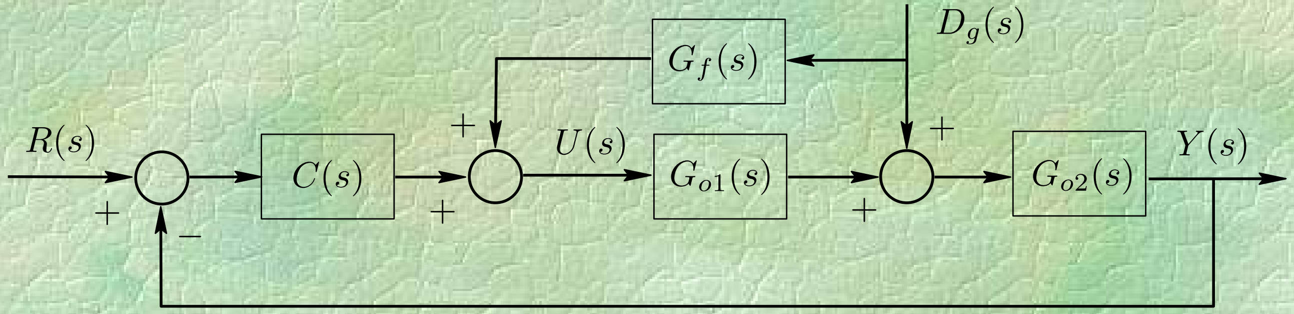


Figure 10.2: *Disturbance feedforward scheme.*

The proposed architecture has the following features

(i) The feedforward block transfer function $G_f(s)$ must be stable and proper, since it acts in open loop.

(ii) Ideally, the feedforward block should invert part of the nominal model, i.e.

$$G_f(s) \cong -[G_{01}(s)]^{-1}$$

(iii) Since usually $G_{01}(s)$ will have a low pass characteristic, we should expect $G_f(s)$ to have a high pass characteristic.

Example of Disturbance Feedforward

Consider a plant having a nominal model given by

$$G_o(s) = \frac{e^{-s}}{2s^2+3s+1} \qquad G_{o1}(s) = \frac{1}{s+1} \qquad G_{o2}(s) = \frac{e^{-s}}{2s+1}$$

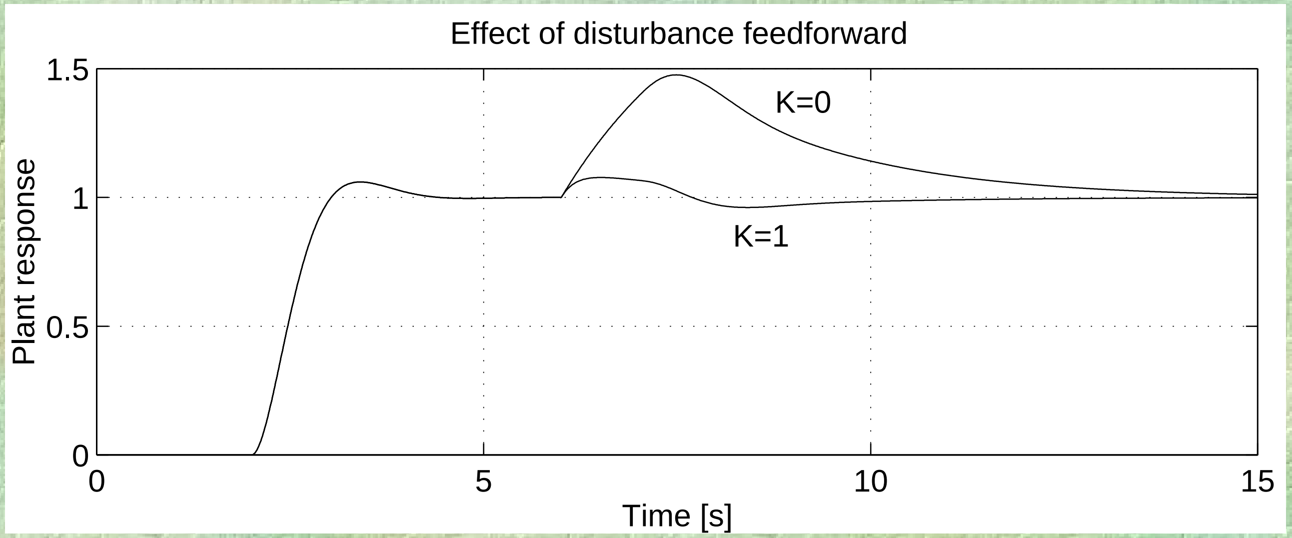
We assume that the disturbance $d_g(t)$ consists of infrequently occurring step changes. A feedback only solution to this problem would be hindered by the fact that the achievable loop bandwidth would be constrained by the presence of the delay in G_0. We therefore investigate the use of feedforward control. We choose the architecture shown earlier in Figure 10.2 and choose $-G_f(s)$ as an approximation to the inverse of $G_{01}(s)$, i.e.

$$G_f(s) = -K\frac{s+1}{\beta s+1}$$

Where β allows a trade off to be made between the effectiveness of the feedforward versus the size of the control effort. Note that K takes the nominal value 1.

The next figure shows the effect of varying K from 0 (no disturbance feedforward) to $K = 1$ (full disturbance feedforward). [A unit step reference is applied at $t = 1$ followed by a unit step disturbance at $t = 5$].

Figure 10.3: *Control loop with (K = 1) and without (K = 0) disturbance feedforward*



We thus see that the use of disturbance feedforward can anticipate the disturbance and lead to significantly improved transient response.

Industrial Application of Feedforward Control

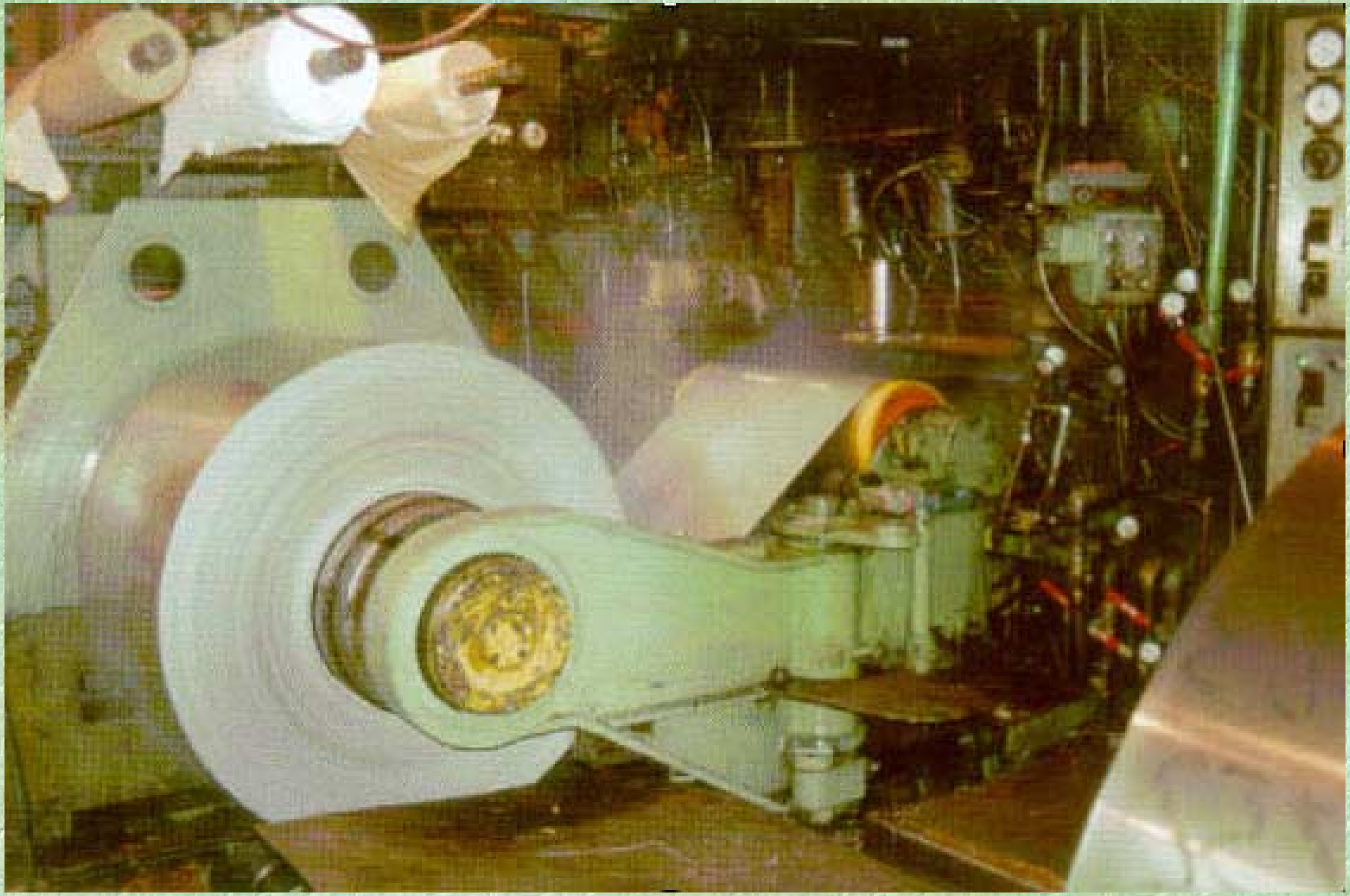
Feedforward control is generally agreed to be one of the most useful concepts in practical control system design beyond the use of elementary feedback ideas.

We will illustrate the idea by revisiting the hold up effect in Rolling Mills which was discussed in Chapter 8.

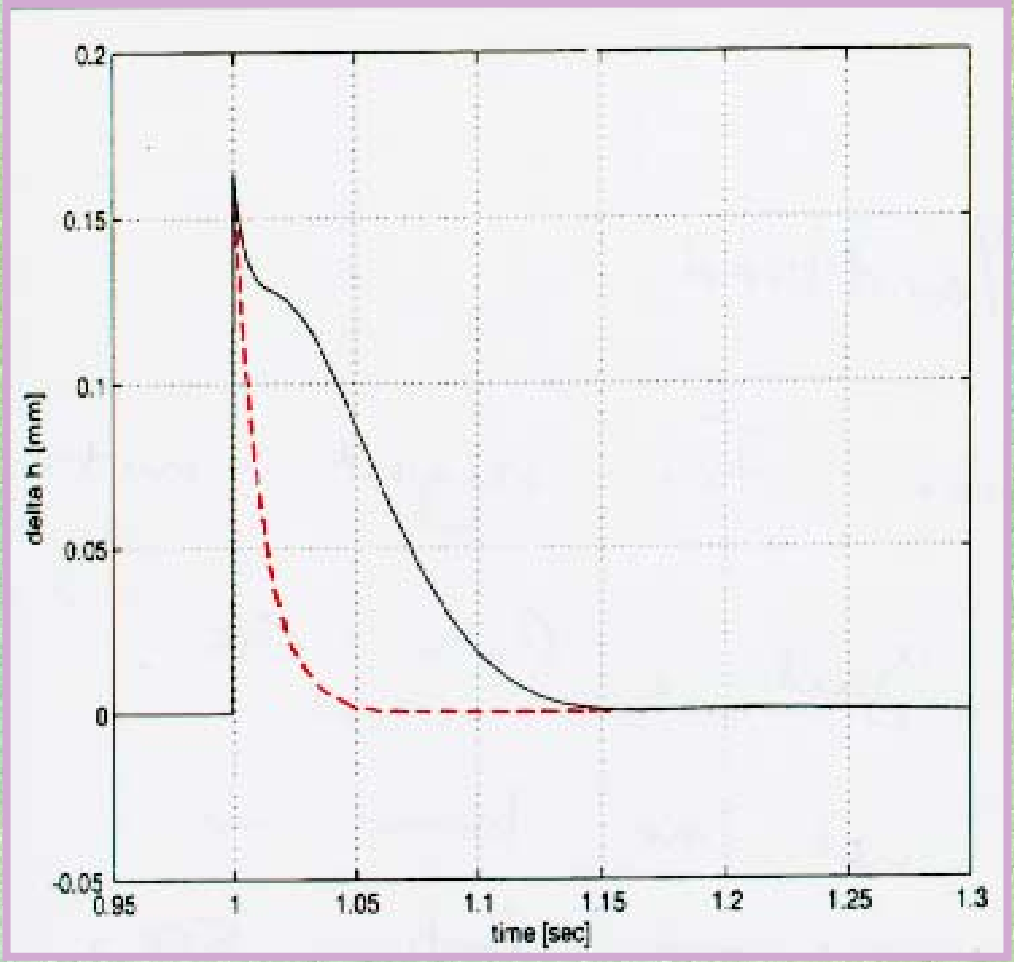
Hold-Up Effect in Reversing Mill Revisited

Consider again the Rolling Mill problem discussed earlier. There we saw that the presence of imaginary axis zeros were a fundamental limitation impeding the achievement of a rapid response between unloaded roll gap position and exit thickness. We called this the *hold-up* effect. The physical origin of the problem is tension interactions.

Reversing Mill



Hold Up Effect



The dotted line represents the expected disturbance response whereas what is actually achieved is the solid line.

Consider the schematic diagram shown on the next slide. We recall that the physical explanation for the hold-up effect is as follows:

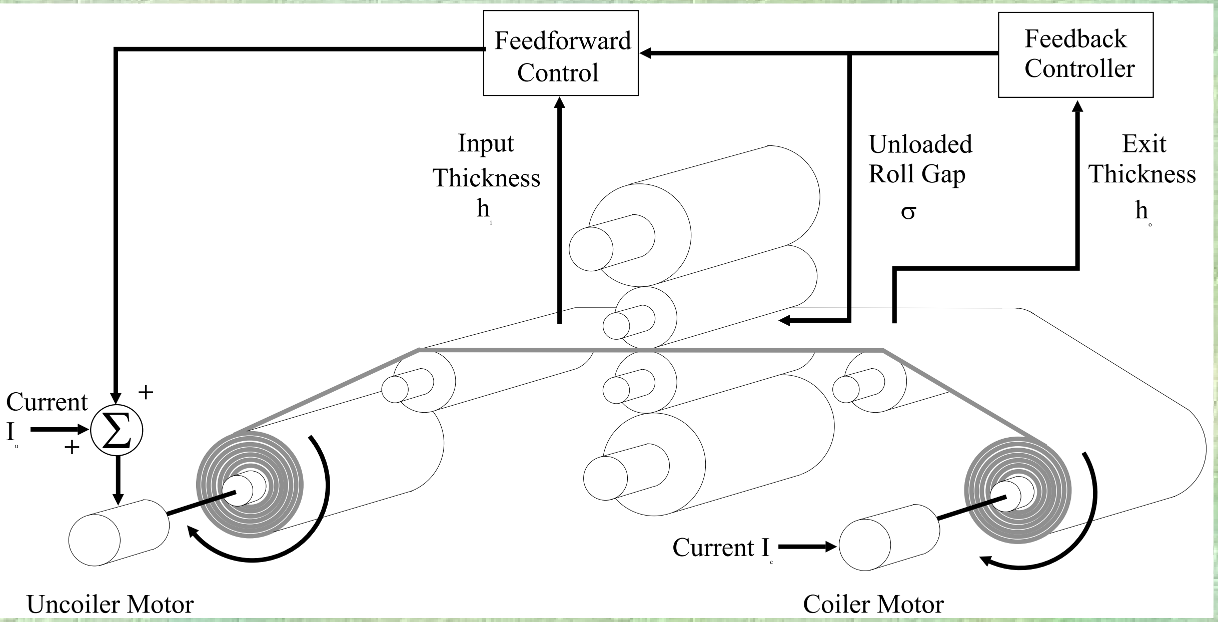
- Say the roll gap is opened;
- Initially this causes the exit thickness to increase;
- However, the exit speed is roughly constant (due to the action of another control loop), hence more mass comes out the end of the mill;
- Hence the incoming strip velocity must increase to supply this extra mass flow;

- However, due to the inertia of the uncoiler, this means that the input tension will increase;
- In turn, increased input tension implies a drop in exit thickness.

The exit thickness increase is thus *held up* until the uncoiler current controller can respond and restore the tension to its original value.

This phenomena manifests itself in the imaginary axis zero noted in Chapter 8 in the model linking roll gap to exit thickness.

Figure 10.6: *Feedforward controller for reversing mill*



The above explanation suggests that a remedy might be to send a pulse of current to the uncoiler motor as soon as we adjust the roll gap, i.e. to use **FEEDFORWARD**.

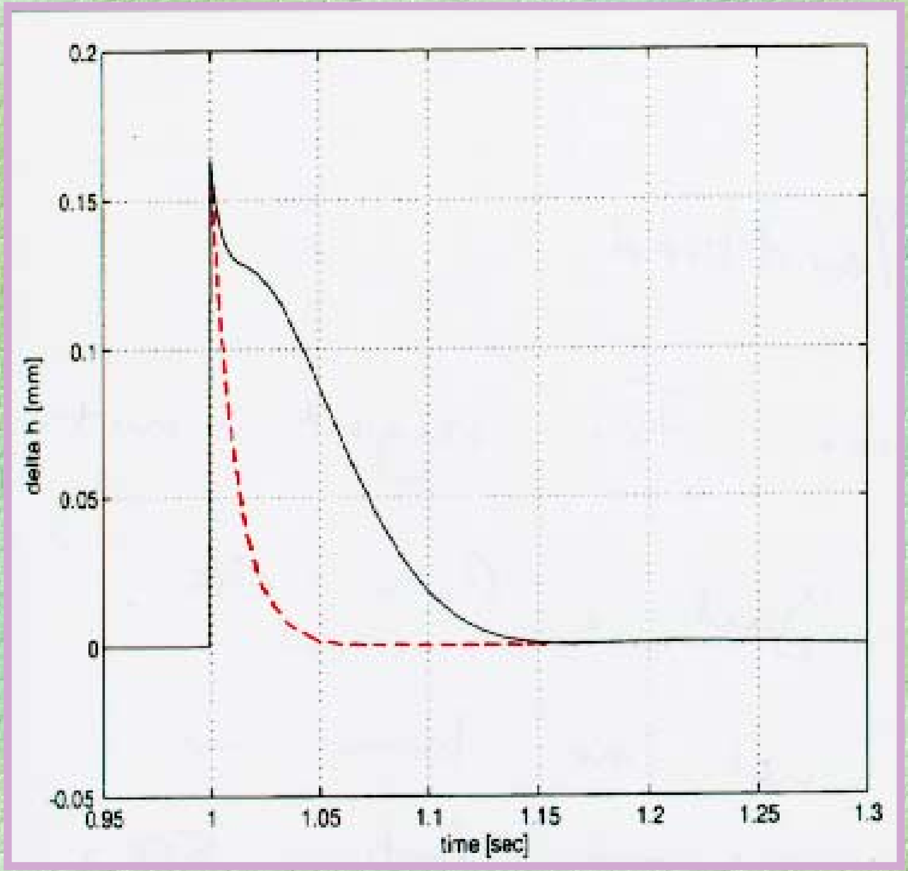
Indeed, one can show using the physics of the problem that tension fluctuations would be avoided by choosing the uncoiler current as

$$i_u(t) = \frac{J_u \omega_u^o}{v_i^o h_i^o K_m} \left[c_1 v_0^o \frac{d\sigma(t)}{dt} + c_2 v_0^o \frac{dh_i(t)}{dt} - v_i^o \frac{dh_i(t)}{dt} \right]$$

The above equation is seen to be a *feedforward* signal linking (*the derivatives of*) the unloaded roll gap position, $\sigma(t)$, and the input thickness, $h_i(t)$, to the uncoiler current.

Use of feedforward control in this example removes the fundamental limitation arising from the imaginary axis zero. This is not a contradiction in terms because the limitation was only fundamental within the single input (*roll gap*) single output (*exit thickness*) architecture. Changing the **architecture** by use of feedforward control to the uncoiler currents alters the fundamental nature of the problem and removes the limitation.

Result with Feedforward Control



Recall that the solid line was the best that could be achieved with a single degree of freedom control whereas using feedforward we can achieve the dotted line.

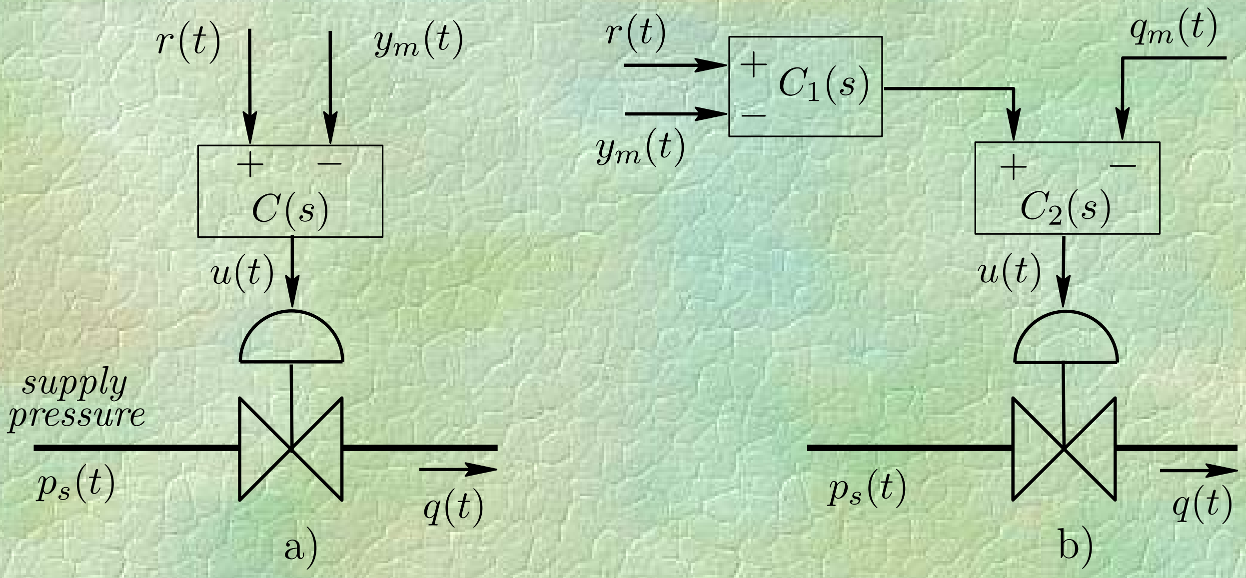
The above example delivers an important message in solving tough control problems. Specifically, one should look out for architectural changes which may dramatically change a difficult (*or maybe impossible*) problem into an easy one.

3. Cascade Control

Next we turn to an alternative architecture for dealing with disturbances. The core idea is to feedback intermediate variables that lie between the disturbance injection point and the output. This gives rise to so called, *cascade control*.

Cascade control is very commonly used in practice. For example, if one has a valve in a control loop, then it is usually a good idea to place a cascade controller around the valve. This requires measurements to be made of the flow out of the valve (*see next slide*) but can significantly improve the overall performance due to the linearizing effect that local feedback around the valve has.

Figure 10.7: *Example of application of cascade control*

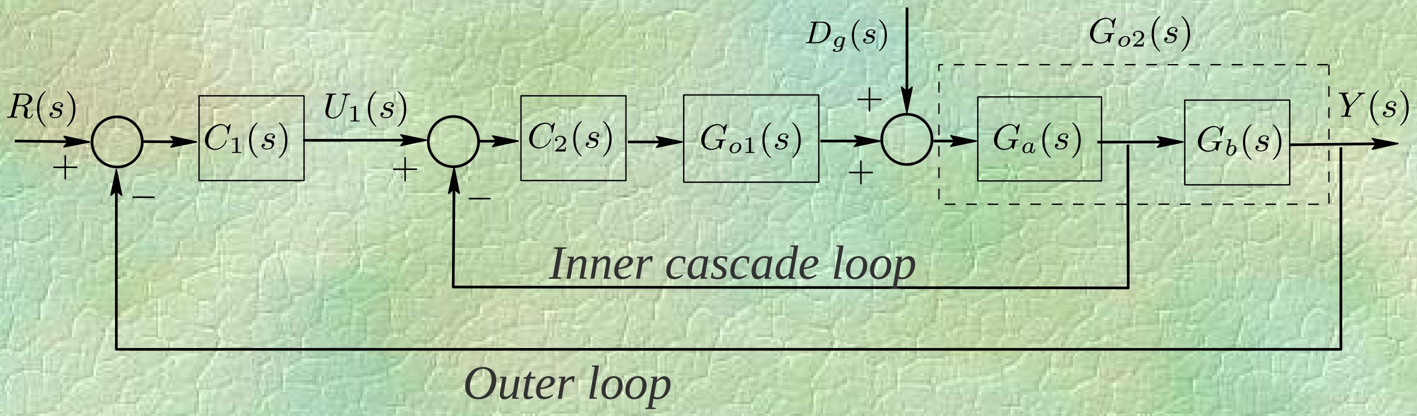


Non-cascade Valve Controller

Cascade Valve Controller

Figure 10.8: *Cascade control structure*

The generalization of this idea has the structure as shown below:



Referring to Figure 10.8 (*previous slide*), the main benefits of cascade control are obtained

(i) when $G_a(s)$ contains significant nonlinearities that limit the loop performance;

or

(ii) when $G_b(s)$ limits the bandwidth in a basic control architecture.

Example of Cadcade Control

Consider a plant having the same nominal model as in the previous example on disturbance feedforward. Assume that the measurement for the secondary loop is the input to $G_{02}(s)$,

$$G_{o1}(s) = \frac{1}{s+1}; \qquad G_{o2}(s) = \frac{e^{-s}}{2s+1}; \qquad G_a(s) = 1; \qquad G_b(s) = G_{o2}(s) = \frac{e^{-s}}{2s+1}$$

We first choose the secondary controller to be a PI controller where

$$C_2(s) = \frac{8(s+1)}{s}$$

This leads to an inner loop having effective closed loop transfer function of

$$T_{o2}(s) = \frac{8}{s+8}$$

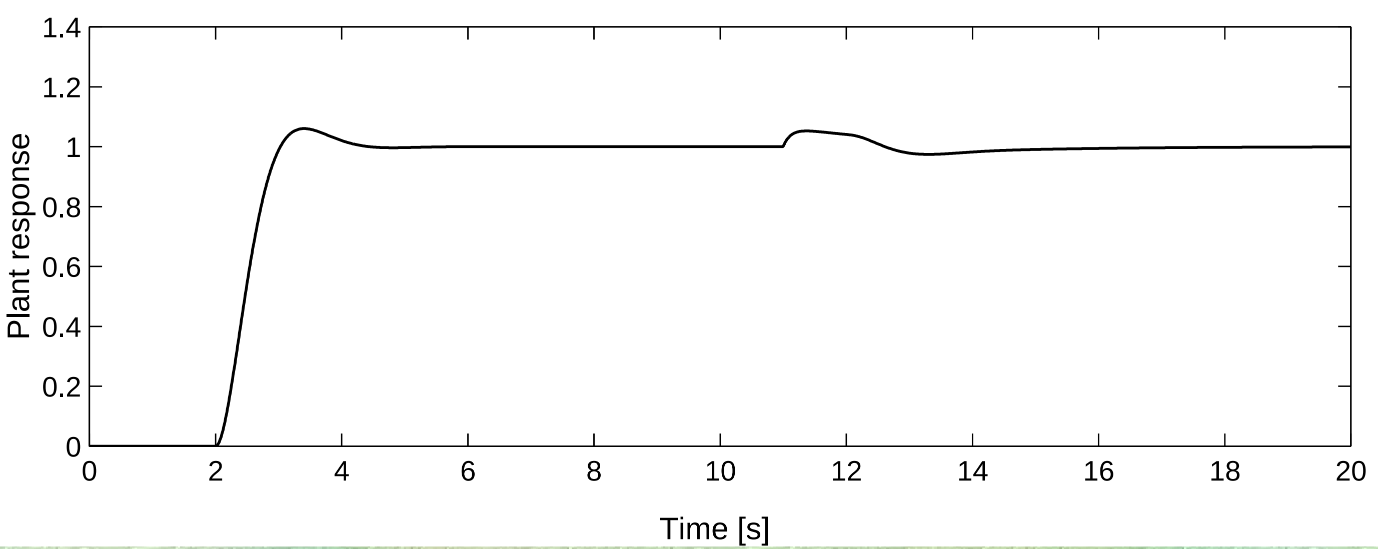
Hence the primary (or outer loop) controller sees an equivalent plant with transfer function

$$G_{oeq}(s) = \frac{8e^{-s}}{2s^2 + 17s + 8}$$

The outter controller is then designed using a Smith Predictor (*see Chapter 7*).

The results for the same disturbance as in the earlier example on disturbance feedforward are shown in the next slide. [A unit step reference is applied at $t = 1$ followed by a unit step disturbance at $t = 5$].

Figure 10.9: *Disturbance rejection with a cascade control loop*



Comparing Figure 10.9 with Figure 10.3 we see that cascade control has achieved similar disturbance rejection (*for this example*) as was achieved earlier using disturbance feedforward.

The main features of cascade control are

(i) Cascade control is a feedback strategy.

(ii) A second measurement of a process variable is required. However, the disturbance itself does not need to be measured. Indeed, the secondary loop can be interpreted as having an observer to estimate the disturbance.

(iii) Measurement noise in the secondary loop must be considered in the design, since it may limit the achievable bandwidth in this loop.

(iv) Although cascade control (in common with feedforward) requires inversion, it can be made less sensitive to modeling errors by using the advantages of feedback.

Summary

- This chapter focuses the discussion of the previous chapter on a number of special topics with high application value:
  - internal disturbance models: compensation for classes of references and disturbances
  - feedforward
  - cascade control
  - two-degree of freedom architectures

Signal models

- Certain classes of reference or disturbance signals can be modeled explicitly by their Laplace transform:

Signal Type	*Transform*
Step	$^{1}/_{s}$
Ramp	$(a_1 s + 1) / s^2$
Parabola	$(a_2 s^2 + a_1 s + 1) / s^3$
Sinusoid	$(a_1 s + 1) / (s^2 + w^2)$

- such references (*disturbances*) can be asymptotically tracked (*rejected*) if and only if the closed loop contains the respective transform in the sensitivity S_0.

- This is equivalent to having imagined the transforms being (unstable) poles of the open-loop and stabilizing them with the controller.
- In summary, the internal model principle augments poles to the open loop gain function $G_0(s)C(s)$. However, this implies that the same design trade-offs apply as if these poles had been in the plant to begin with.
- Thus internal model control is not cost free but must be considered as part of the design trade-off considerations.

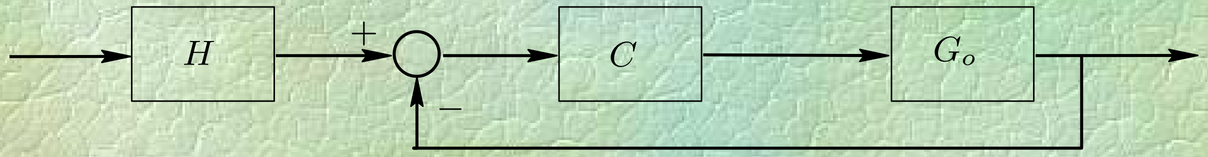
Reference feedforward

- A simple but very effective technique for improving responses to setpoint changes is prefiltering the setpoint (*see next slide*).

- This is the so called two-degree-of-freedom (two d.o.f.) architecture since the prefilter *H* provides an additional design freedom. If, for example, there is significant measurement noise, then the loop must not be designed with too high a bandwidth. In this situation, reference tracking can be sped up with the prefilter.

- Also, if the reference contains high-frequency components (*such as step changes*, *for example*), which are anyhow beyond the bandwidth of the loop, then one might as well filter them so not to excite uncertainties and actuators with them unnecessarily.
- It is important to note, however, that design inadequacies in the loop (*such as poor stability or performance*) cannot be compensated by the prefilter. This is due to the fact that the prefilter does not affect the loop dynamics excited by disturbances.

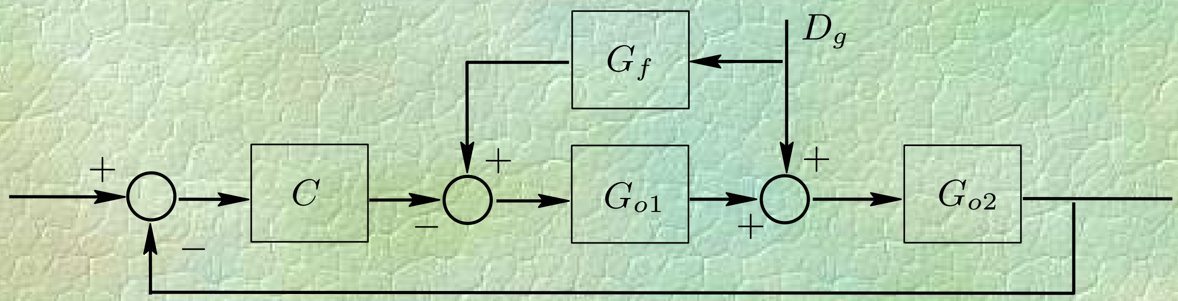
Figure 10.10: *Two degree of freedom architecture for improved tracking*



- Disturbance feedforward
  - The trade-offs regarding sensitivities to reference, measurement noise, input- and output disturbances as discussed in the previous chapters refer to the case when these disturbances are technically or economically not measureable.

    Measurable disturbances can be compensated for explicitly by disturbance feedforward (*see next slide*) thus relaxing one of the trade-off constraints and giving the design more flexibility.

Figure 10.11: *Disturbance feedforward structure*



Cascade Control

- Cascade control is another well-proven technique applicable when two or more systems feed sequentially into each other (*see next slide*).
- All previously discussed design trade-offs and insights apply.
- If the inner loop (C_2 *in Figure 10.12*) were not utilized, then the outer controller (C_1 in *Figure 10.12*) would implicitly or explicitly estimate y1 as an internal state of the overall system ($G_{01}G_{02}$). This estimate, however, would inherit the model uncertainty associated with $G0_2$. Therefore, utilizing the available measurement of y_1 reduces the overall uncertainty and one can achieve the associated benefits.

Figure 10.12: *Cascade control structure*