

Chapter 6

Classical PID Control

This chapter examines a particular control structure that has become almost universally used in industrial control. It is based on a particular fixed structure controller family, the so-called PID controller family. These controllers have proven to be robust and extremely beneficial in the control of many important applications.

PID stands for: **P** (*Proportional*)

I (*Integral*)

D (*Derivative*)

Historical Note

Early feedback control devices implicitly or explicitly used the ideas of proportional, integral and derivative action in their structures. However, it was probably not until Minorsky's work on ship steering* published in 1922, that rigorous theoretical consideration was given to PID control.

This was the first mathematical treatment of the type of controller that is now used to control almost all industrial processes.

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* Minorsky (1922) "Directional stability of automatically steered bodies", *J. Am. Soc. Naval Eng.*, 34, p.284.

The Current Situation

Despite the abundance of sophisticated tools, including advanced controllers, the Proportional, Integral, Derivative (PID controller) is still the most widely used in modern industry, controlling more that 95% of closed-loop industrial processes*

* Åström K.J. & Hägglund T.H. 1995, "New tuning methods for PID controllers", *Proc. 3rd European Control Conference*, p.2456-62; and

Yamamoto & Hashimoto 1991, "Present status and future needs: The view from Japanese industry", Chemical Process Control, *CPCIV, Proc. 4th International Conference on Chemical Process Control*, Texas, p.1-28.

PID Structure

Consider the simple SISO control loop shown in Figure 6.1:

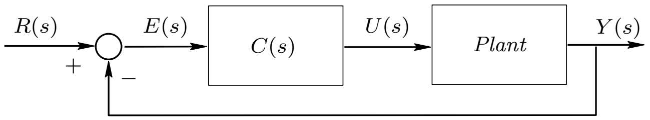


Figure 6.1: *Basic feedback control loop*

The *standard form* PID are:

Proportional only: $$C_P(s) = K_p$$

Proportional plus Integral: $$C_{PI}(s) = K_p\left(1 + \frac{1}{T_r s}\right)$$

Proportional plus derivative: $$C_{PD}(s) = K_p\left(1 + \frac{T_d s}{\tau_D s + 1}\right)$$

Proportional, integral and derivative: $$C_{PID}(s) = K_p\left(1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_D s + 1}\right)$$

An alternative *series* form is:

$$C_{series}(s) = K_s \left(1 + \frac{I_s}{s}\right)\left(1 + \frac{D_s s}{\gamma_s D_s s + 1}\right)$$

Yet another alternative form is the, so called, parallel form:

$$C_{parallel}(s) = K_p + \frac{I_p}{s} + \frac{D_p s}{\gamma_p D_p s + 1}$$

Tuning of PID Controllers

Because of their widespread use in practice, we present below several methods for tuning PID controllers. Actually these methods are quite old and date back to the 1950's. Nonetheless, they remain in widespread use today.

In particular, we will study.

- *Ziegler-Nichols Oscillation Method*
- *Ziegler-Nichols Reaction Curve Method*
- *Cohen-Coon Reaction Curve Method*

(1) Ziegler-Nichols (Z-N) Oscillation Method

This procedure is only valid for open loop stable plants and it is carried out through the following steps

- Set the true plant under proportional control, with a very small gain.
- Increase the gain until the loop starts oscillating. Note that linear oscillation is required and that it should be detected at the controller output.

- Record the controller critical gain $K_p = K_c$ and the oscillation period of the controller output, P_c.
- Adjust the controller parameters according to Table 6.1 (*next slide*); there is some controversy regarding the PID parameterization for which the Z-N method was developed, but the version described here is, to the best knowledge of the authors, applicable to the parameterization of standard form PID.

Table 6.1: *Ziegler-Nichols tuning using the oscillation method*

	K_p	T_r	T_d
P	$0.50K_c$		
PI	$0.45K_c$	$\frac{P_c}{1.2}$	
PID	$0.60K_c$	$0.5P_c$	$\frac{P_c}{8}$

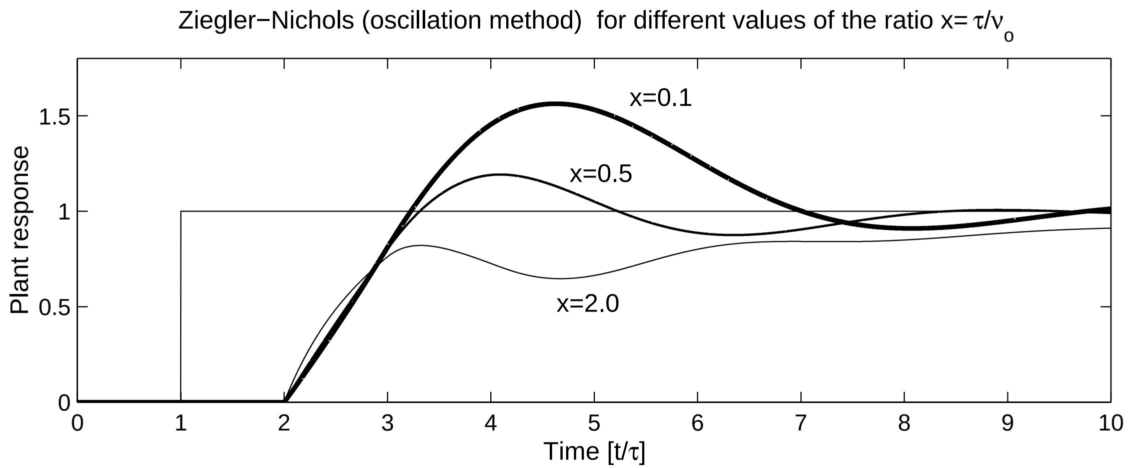
General System

If we consider a general plant of the form:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{\gamma_0 s + 1}; \quad \gamma_0 > 0$$

then one can obtain the PID settings via Ziegler-Nichols tuning for different values of τ and ν_0. The next plot shows the resultant closed loop step responses as a function of the ratio $x \stackrel{\Delta}{=} \frac{\tau}{\nu_0}$.

Figure 6.3: *PI Z-N tuned (oscillation method) control loop for different values of the ratio* $x \stackrel{\Delta}{=} \frac{\tau_0}{\nu_0}$.



Numerical Example

Consider a plant with a model given by

$$G_o(s) = \frac{1}{(s+1)^3}$$

Find the parameters of a PID controller using the Z-N oscillation method. Obtain a graph of the response to a unit step input reference and to a unit step input disturbance.

Solution

Applying the procedure we find:

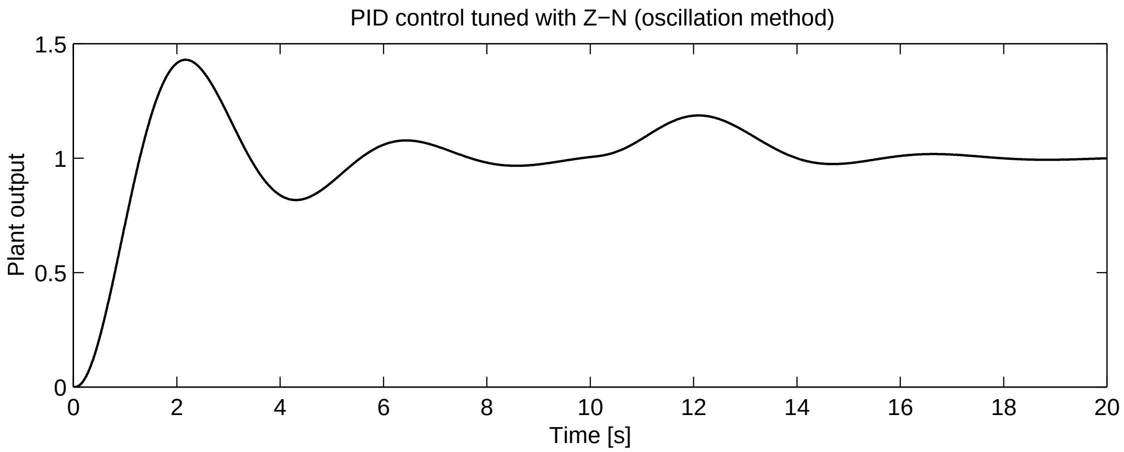
$$K_c = 8 \text{ and } \omega_c = \sqrt{3}.$$

Hence, from Table 6.1, we have

$$K_p = 0.6 * K_c = 4.8; \qquad T_r = 0.5 * P_c \approx 1.81; \qquad T_d = 0.125 * P_c \approx 0.45$$

The closed loop response to a unit step in the reference at $t = 0$ and a unit step disturbance at $t = 10$ are shown in the next figure.

Figure 6.4: *Response to step reference and step input disturbance*



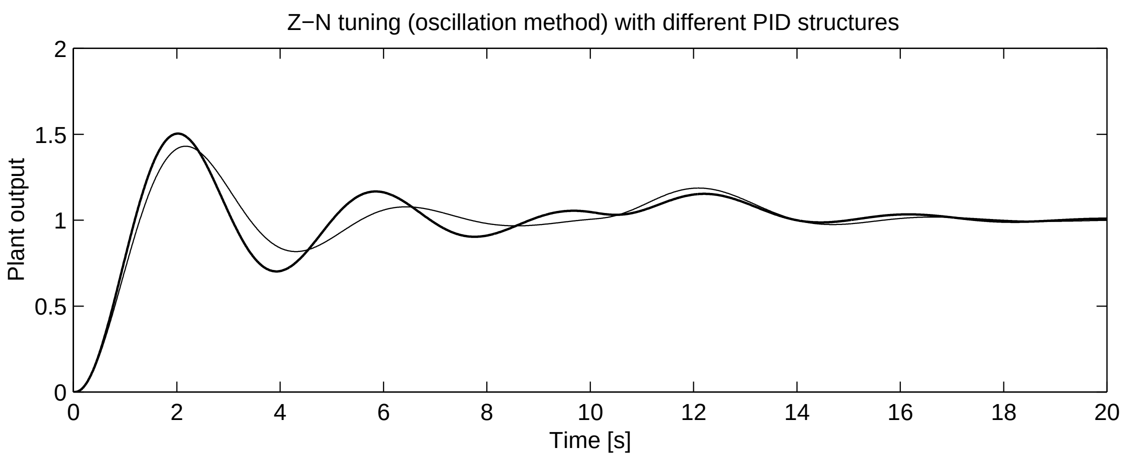
Different PID Structures?

A key issue when applying PID tuning rules (such as Ziegler-Nichols settings) is that of which PID structure these settings are applied to.

To obtain an appreciation of these differences we evaluate the PID control loop for the same plant in Example 6.1, but with the Z-N settings applied to the series structure, i.e. in the notation used in (6.2.5), we have

$$K_s = 4.8 \quad I_s = 1.81 \quad D_s = 0.45 \quad \gamma_s = 0.1$$

Figure 6.5: *PID Z-N settings applied to series structure (thick line) and conventional structure (thin line)*



Observation

In the above example, it has not made much difference, to which form of PID the tuning rules are applied. However, the reader is warned that this can make a difference in general.

(2) Reaction Curve Based Methods

A linearized quantitative version of a simple plant can be obtained with an open loop experiment, using the following procedure:

1. With the plant in open loop, take the plant manually to a normal operating point. Say that the plant output settles at $y(t) = y_0$ for a constant plant input $u(t) = u_0$.
2. At an initial time, t_0, apply a step change to the plant input, from u_0 to u_∞ (*this should be in the range of 10 to 20% of full scale*).

Cont/...

3. Record the plant output until it settles to the new operating point. Assume you obtain the curve shown on the next slide. This curve is known as the *process reaction curve*.

   In Figure 6.6, m.s.t. stands for *maximum slope tangent*.

4. Compute the parameter model as follows

$$K_o = \frac{y_\infty - y_o}{u_\infty - u_o}; \qquad \tau_o = t_1 - t_o; \qquad \nu_o = t_2 - t_1$$

Figure 6.6: *Plant step response*

The suggested parameters are shown in Table 6.2.

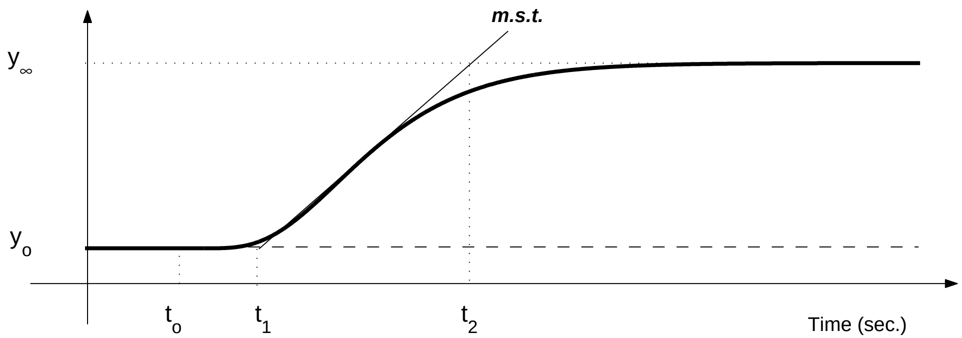


Table 6.2: *Ziegler-Nichols tuning using the reaction curve*

	$\mathbf{K_p}$	$\mathbf{T_r}$	$\mathbf{T_d}$
P	$\frac{\nu_o}{K_o\tau_o}$		
PI	$\frac{0.9\nu_o}{K_o\tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o\tau_o}$	$2\tau_o$	$0.5\tau_o$

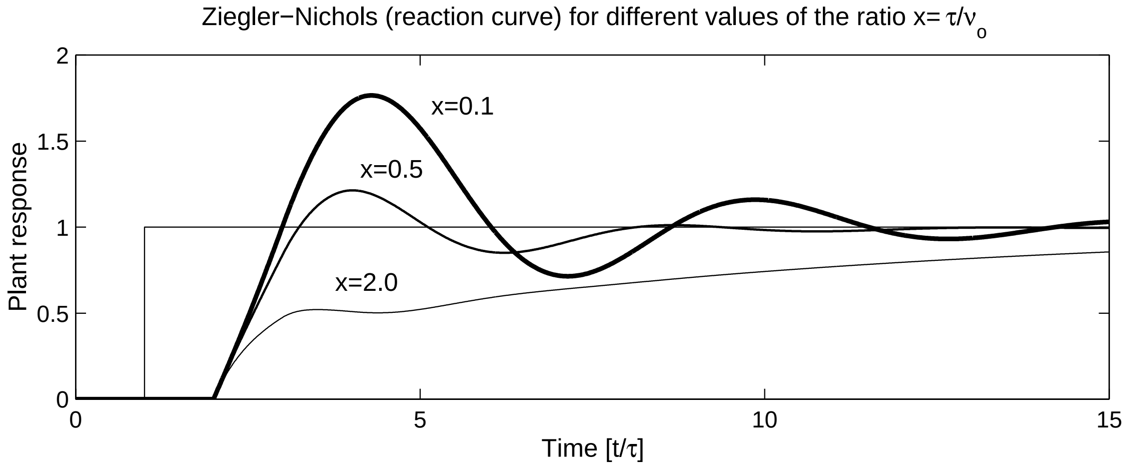
General System Revisited

Consider again the general plant:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{\gamma_0 s + 1}$$

The next slide shows the closed loop responses resulting from Ziegler-Nichols Reaction Curve tuning for different values of $x \stackrel{\Delta}{=} \frac{\tau}{\nu_0}$.

Figure 6.7: *PI Z-N tuned (reaction curve method) control loop*



Observation

We see from the previous slide that the Ziegler-Nichols reaction curve tuning method is very sensitive to the ratio of delay to time constant.

(3) Cohen-Coon Reaction Curve Method

Cohen and Coon carried out further studies to find controller settings which, based on the same model, lead to a weaker dependence on the ratio of delay to time constant. Their suggested controller settings are shown in Table 6.3:

	$\mathbf{K_p}$	$\mathbf{T_r}$	$\mathbf{T_d}$
P	$\frac{\nu_o}{K_o\tau_o}\left[1+\frac{\tau_o}{3\nu_o}\right]$		
PI	$\frac{\nu_o}{K_o\tau_o}\left[0.9+\frac{\tau_o}{12\nu_o}\right]$	$\frac{\tau_o[30\nu_o+3\tau_o]}{9\nu_o+20\tau_o}$	
PID	$\frac{\nu_o}{K_o\tau_o}\left[\frac{4}{3}+\frac{\tau_o}{4\nu_o}\right]$	$\frac{\tau_o[32\nu_o+6\tau_o]}{13\nu_o+8\tau_o}$	$\frac{4\tau_o\nu_o}{11\nu_o+2\tau_o}$

Table 6.3: *Cohen-Coon tuning using the reaction curve.*

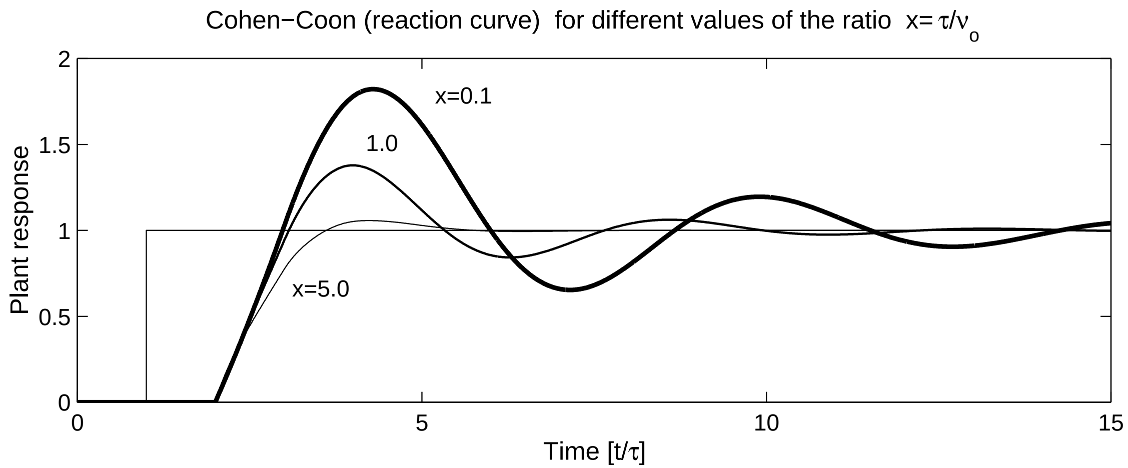
General System Revisited

Consider again the general plant:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{\gamma_0 s + 1}$$

The next slide shows the closed loop responses resulting from Cohen-Coon Reaction Curve tuning for different values of $x = \frac{\tau}{\nu_0}$.

Figure 6.8: *PI Cohen-Coon tuned (reaction curve method) control loop*



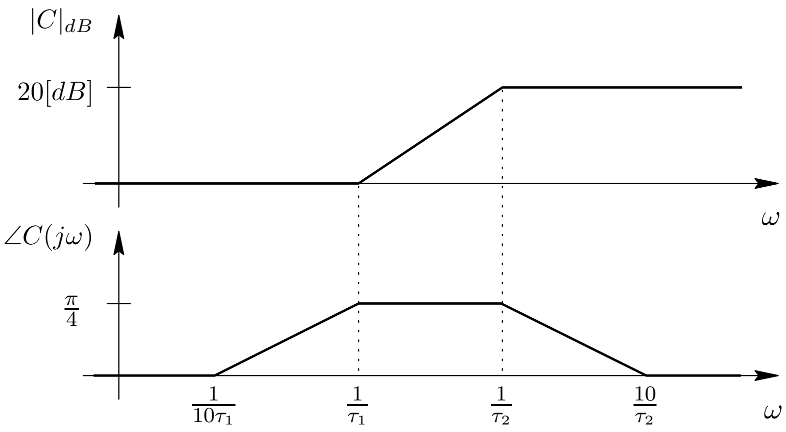
Lead-lag Compensators

Closely related to PID control is the idea of lead-lag compensation. The transfer function of these compensators is of the form:

$$C(s) = \frac{\tau_1 s + 1}{\tau_2 s + 1}$$

If $\tau_1 > \tau_2$, then this is a *lead network* and when $\tau_1 < \tau_2$, this is a *lag network*.

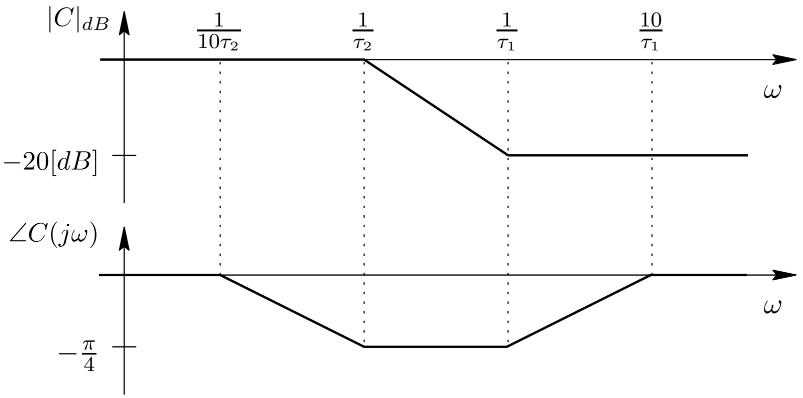
Figure 6.9: *Approximate Bode diagrams for lead networks (τ_1=10τ_2)*



Observation

We see from the previous slide that the lead network gives phase advance at $\omega = 1/\tau_1$ without an increase in gain. Thus it plays a role similar to derivative action in PID.

Figure 6.10: *Approximate Bode diagrams for lag networks ($\tau_2 = 10\tau_1$)*



Observation

We see from the previous slide that the lag network gives low frequency gain increase. Thus it plays a role similar to integral action in PID.

Illustrative Case Study: *Distillation Column*

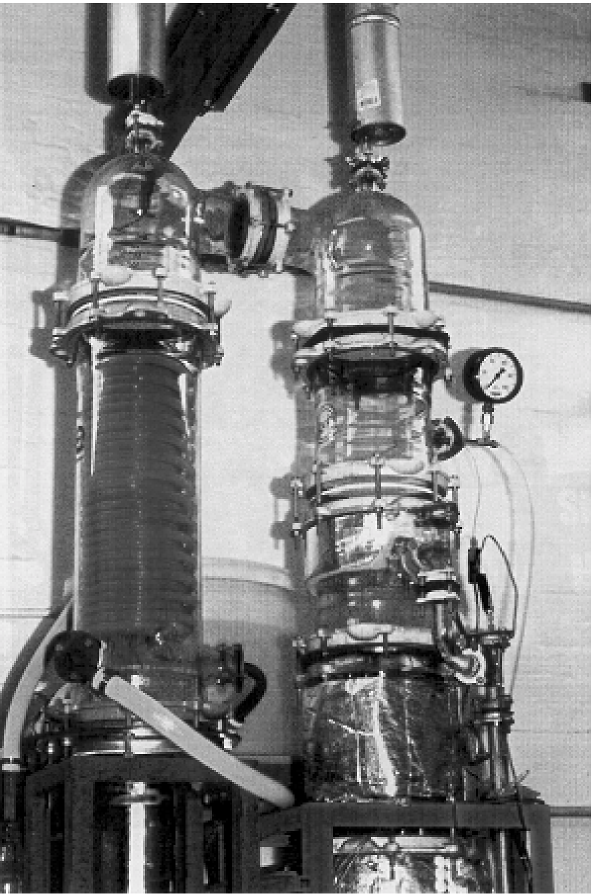
PID control is very widely used in industry. Indeed, one would we hard pressed to find loops that do not use some variant of this form of control.

Here we illustrate how PID controllers can be utilized in a practical setting by briefly examining the problem of controlling a distillation column.

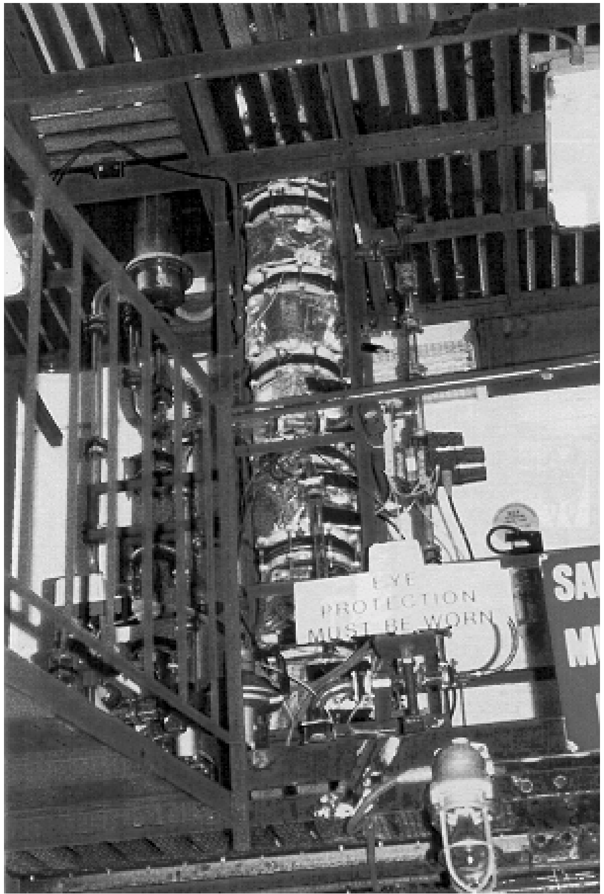
Example System

The specific system we study here is a pilot scale ethanol-water distillation column. Photos of the column (*which is in the Department of Chemical Engineering at the University of Sydney, Australia*) are shown on the next slide.

Condenser



Feed-point



Reboiler

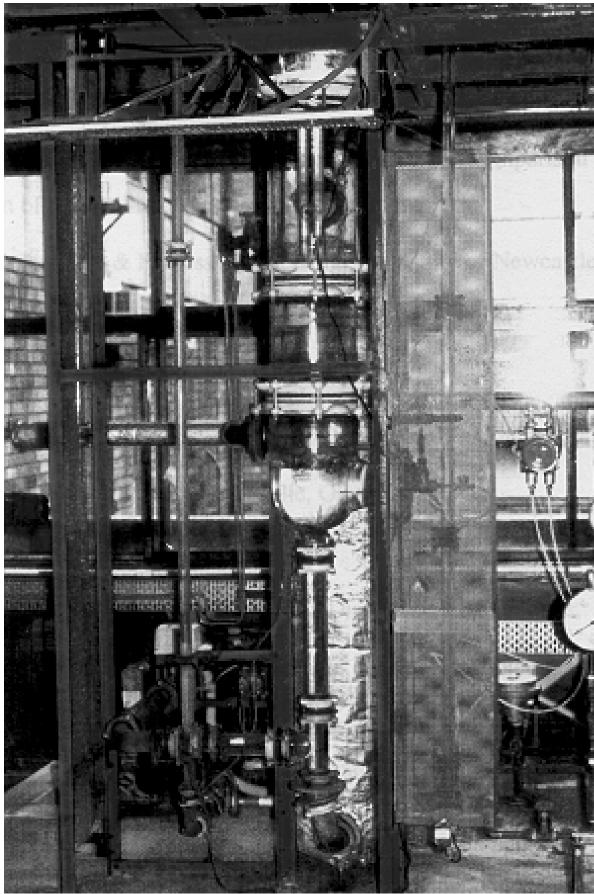
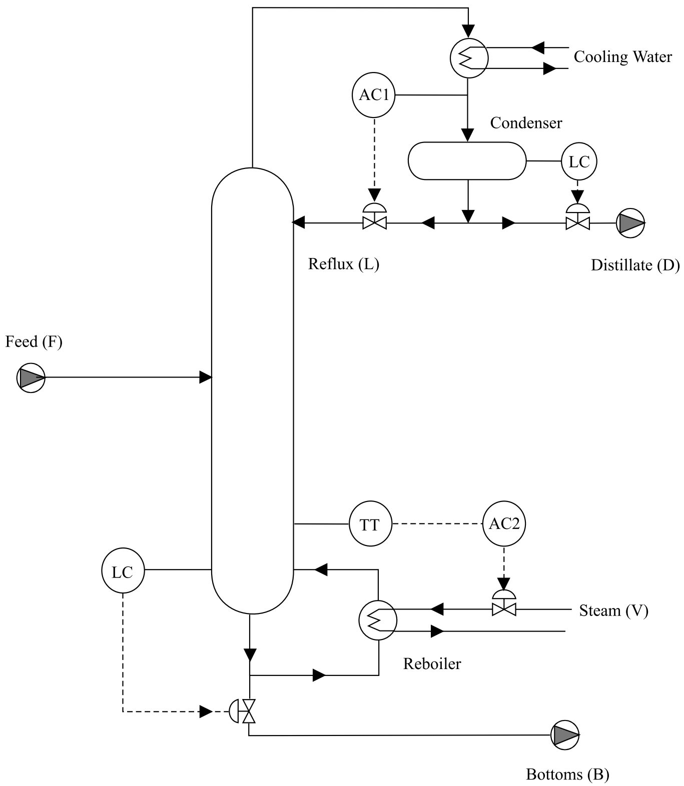


Figure 6.11: *Ethanol - water distillation column*

A schematic diagram of the column is given below:



Model

A locally linearized model for this system is as follows:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

where

$$G_{11}(s) = \frac{0.66e^{-2.6s}}{6.7s+1}$$

$$G_{12}(s) = \frac{-0.0049e^{-s}}{9.06s+1}$$

$$G_{21}(s) = \frac{-34.7e^{-9.2s}}{8.15s+1}$$

$$G_{22}(s) = \frac{0.87(11.6s+1)e^{-s}}{(3.89s+1)(18.8s+1)}$$

Note that the units of time here are minutes.

Decentralized PID Design

We will use two PID controllers:

One connecting Y_1 to U_1

The other, connecting Y_2 to U_2 .

In designing the two PID controllers we will initially ignore the two transfer functions G_{12} and G_{21}. This leads to two separate (and non-interacting) SISO systems. The resultant controllers are:

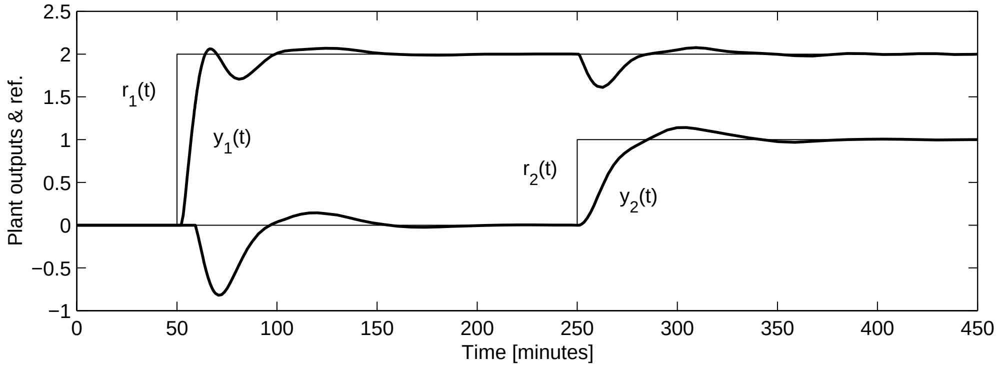
$$C_1(s) = 1 + \frac{0.25}{s}$$
$$C_s(s) = 1 + \frac{0.15}{s}$$

We see that these are of PI type.

Simulations

We simulate the performance of the system with the two decentralized PID controllers. A two unit step in reference 1 is applied at time $t = 50$ and a one unit step is applied in reference 2 at time $t = 250$. The system was simulated with the true coupling (i.e. including G_{12} and G_{21}). The results are shown on the next slide.

Figure 6.12: Simulation results for PI control of distillation column



It can be seen from the figure that the PID controllers give quite acceptable performance on this problem. However, the figure also shows something that is very common in practical applications - namely the two loops interact i.e. a change in reference r_1 not only causes a change in y_1 (as required) but also induces a transient in y_2. Similarly a change in the reference r_2 causes a change in y_2 (as required) and also induces a change in y_1. In this particular example, these interactions are probably sufficiently small to be acceptable. Thus, in common with the majority of industrial problems, we have found that two simple PID (actually PI in this case) controllers give quite acceptable performance for this problem. Later we will see how to design a full multivariable controller for this problem that accounts for the interaction.

Summary

- PI and PID controllers are widely used in industrial control.
- From a modern perspective, a PID controller is simply a controller of (up to second order) containing an integrator. Historically, however, PID controllers were tuned in terms of their **P**, **I** and **D** terms.
- It has been empirically found that the PID structure often has sufficient flexibility to yield excellent results in many applications.

- The basic term is the proportional term, **P**, which causes a corrective control actuation proportional to the error.

- The integral term, **I** gives a correction proportional to the integral of the error. This has the positive feature of ultimately ensuring that sufficient control effort is applied to reduce the tracking error to zero. However, integral action tends to have a destabilizing effect due to the increased phase shift.

- The derivative term, **D**, gives a predictive capability yielding a control action proportional to the rate of change of the error. This tends to have a stabilizing effect but often leads to large control movements.

- Various empirical tuning methods can be used to determine the PID parameters for a given application. They should be considered as a first guess in a search procedure.

- Attention should also be paid to the PID structure.
- Systematic model-based procedures for PID controllers will be covered in later chapters.
- A controller structure that is closely related to PID is a lead-lag network. The lead component acts like **D** and the lag acts like **I**.

Useful Sites

The following internet sites give valuable information about PLC's:

www.plcs.net

www.plcopen.org

For example, the next slide lists the manufacturers quoted at the above sites.

ABB
Alfa Laval
Allen-Bradley
ALSTOM/Cegelec
Aromat
Automation Direct/PLC Direct/Koyo/

B&R Industrial Automation
Berthel gmbh

Cegelec/ALSTROM
Control Microsystems
Couzet Automatismes
Control Technology Corporation
Cutler Hammer/IDT

Divelbiss

EBERLE gmbh
Elsag Bailey
Entertron

Festo/Beck Electronic
Fisher & Paykel
Fuji Electric

GE-Fanuc
Gould/Modicon
Grayhill
Groupe Schneider

Cont/....

Hima
Hitachi
Honeywell
Horner Electric

Idec
IDT/Cutler Hammer

Jetter gmbh

Keyence
Kirchner Soft
Klockner-Moeller
Koyo/Automation Direct/PLC Direct

Microconsultants
Mitsubishi
Modicon/Gould
Moore Products

Omron
Opto22

Pilz
PLC Direct/Koyo/Automation Direct

Reliance
Rockwell Automation
Rockwell Software

Cont/….

SAIA-Burgess
Schleicher
Schneider Automation
Siemens
Sigmatek
SoftPLC/Tele-Denken
Square D

Tele-Denken/Soft PLC
Telemecanique
Toshiba
Triangle Research

Z-World

Additional Notes: *Examples commercially available PID controllers*

In the next few slides we briefly describe some of the commercially available PID controllers. There are, of course, a great many such controllers. The examples we have chosen are selected randomly to illustrate the kinds of things that are available.

There are several variations in algorithms, with the three main types being series, parallel and ideal form.

Some controllers are configured to act on the error and some apply the **D** term to the feedback only. Most have special features to deal with saturation and slew rate limits on the plant input. (*This topic is discussed in Chapter 11*).

Allen Bradley PLC-5 PID Block

The PID function in this controller is an output instruction that must be executed periodically at specified intervals determined by the external code.

There are 4 different forms of the controller equation:

(1) With derivative action on the output

$$u = K_c\left\{\left[1+\frac{1}{T_i s}\right]e+\left[\frac{T_d s}{1+\frac{T_d}{16}s}\right]y\right\}+bias$$

(2) With derivative action in the error

$$u = K_c\left\{\left[1+\frac{1}{T_i s}\right]+\left[\frac{T_d s}{1+\frac{T_d}{16}s}\right]\right\}e+bias$$

(3) Similar to (1) but with different gains

$$u = \left[K_p + \frac{K_i}{s} \right] e + \left[\frac{K_d s}{1 + \frac{K_d s}{16 K_p}} \right] y + bias$$

(4) Similar to (2) but with different gains

$$u = \left[K_p + \frac{K_i}{s} + \frac{K_d s}{1 + \frac{K_d s}{16 K_p}} \right] e + bias$$

GEM 80 PIDABS Block

The GEM family of PLC's have a PID block which must be executed periodically at specified intervals determined by the external code. This function is implemented by a velocity type algorithm, with the controller being converted to an absolute controller by adding the previous output value. Thus the controller output is of the form:

$$u_t = u_{t-1} + \frac{P_c\left(e_t - e_{t-1}\right) + I_c e_t + D_c\left(e_t - 2e_{t-1} + e_{t-2}\right)}{100}$$

The reader can convert the above discrete implementation to approximate continuous time form by noting that

$$\frac{e_t - e_{t-1}}{\Delta} \cong \frac{de}{dt}$$

$$\frac{e_t - 2e_{t-1} + e_{t-2}}{\Delta^2} \cong \frac{d^2e}{dt^2}$$

where Δ is the sampling interval. Thus the control law is roughly equivalent to the following:

$$su = \frac{1}{100}\left\{P_c se + \frac{I_c e}{\Delta} + D_c \Delta s^2 e\right\}$$

Two comments regarding this equation are:

(1) Much more will be said on the relationship between $\delta e = \frac{e_t - e_{t-1}}{\Delta}$ and $\frac{de}{dt}$ and Chapters 12, 13 and 14.

(2) Note that to achieve approximately the same performance with different sampling rates, I_c and D_c need to be scaled.

Yokogawa DCS Function Block

This DCS offers nine types of regulatory control blocks -

- *PID*
- *Sampling PI*
- *PID with batch switch*
- *two position on/off controller*
- *three position on/off controller*
- *time proportioning on/off controller*
- *PD with manual reset*
- *blending PI*
- *self tuning PID*

The basic PID controller has 5 variations. The main 3 structures being:

(1) $$u_t = u_{t-1} + K_s\left[K_p\left(e_t - e_{t-1}\right) + \frac{\Delta}{T_i}e_t + \frac{T_d}{\Delta}\left(e_t - 2e_{t-1} + e_{t-2}\right)\right]$$

(2) $$u_t = u_{t-1} + K_s\left[K_p\left(e_t - e_{t-1}\right) + \frac{\Delta}{T_i}e_t + \frac{T_d}{\Delta}\left(y_t - 2y_{t-1} + y_{t-2}\right)\right]$$

(3) $$u_t = u_{t-1} + K_s\left[K_p\left(y_t - y_{t-1}\right) + \frac{\Delta}{T_i}e_t + \frac{T_d}{\Delta}\left(y_t - 2y_{t-1} + y_{t-2}\right)\right]$$

Note that the parameters in these controllers are (*roughly*) invariant w.r.t. Δ.

Additional features of these controllers are

- Selection of the type of equation, including the facility to invert the output;
- Automatic or manual mode selection, with an option for tracking;
- Bumpless transfer;
- Separate input and output limits, including rate and absolute limits;
- Additional non-linear scaling of the output;
- Integrator anti-windup (*called reset-limiter*);
- Selectable execution interval as a multiple of scan time;
- Feed forward, either to the feedback or controller output;
- A dead-band on the controller output.

Fisher Controls 4195K Gauge Pressure Controller

This pressure controller is a pneumatic device, with mechanical linkages, that is coupled to a control valve, specifically for providing pressure regulation. One advantage of pneumatic controllers is that, as they are powered by instrument air, there is no electrical power employed.

The controller can be configured as a P, PI or PID controller, which can be configured as direct or reverse acting. Features such as anti-windup are optional.